Evaluate each index below. No calculators.

Find the odd one out in each row

$\frac{1}{2^{-1}}$	$16^{\frac{1}{4}}$	$4^{\frac{1}{2}}$	$8^{-\frac{1}{3}}$
$16^{\frac{3}{2}}$	$\left(\frac{1}{8}\right)^{-2}$	$(2^3)^2$	4^{-3}
$16^{\frac{3}{4}}$	$\left(\frac{1}{2}\right)^{-3}$	$8^{\frac{2}{3}}$	$\frac{4^2 \times 2^3}{2^4}$
$x^{\frac{3}{2}}$	$x\sqrt{x}$	$\left(\sqrt{x}\right)^3$	$(x^3)^{-\frac{1}{2}}$

F Exponentials and logarithms

F3

Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$.

Know and use the function $\ln x$ and its graph.

Know and use $\ln x$ as the inverse function of e^x

Students should:

- understand and be able to use the equivalences: $y = a^x \Leftrightarrow \log_a y = x$ and $y = e^x \Leftrightarrow \ln y = x$
- know that the graph of $y = \ln x$ is a reflection in the line $y = x$ of the graph of $y = e^x$
- be able to perform simple single transformations (as defined in section B9) of the functions $y = e^x$ and $y = \ln x$
- be able to manipulate logs and exponentials within the solution to a problem.

F Exponentials and logarithms

F5 Solve equations of the form $a^x = b$.

Students should be able to solve equations of the form $a^x = b$, including $e^x = b$

Notes

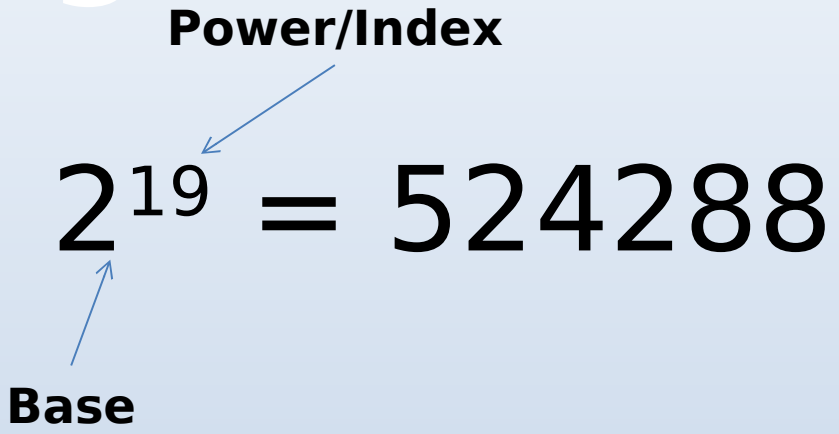
- Equations of this form may require exact answers.
- If exact answers are not required such equations may be solved using a calculator, unless instructions are given to the contrary.

5.1 The laws of logarithms

Solve $2^x = 524288$ $x = 19$

We can do this more efficiently if we use LOGS.

5.1 The laws of logarithms



We say “2 to the power of 19, is 524288”

$\log_2 524288 = ?$

This means what power of 2 gives 524288?

say “log to the base 2 of 524288 is 19”

i.e. you have made the power the subject of the equa

5.1 The laws of logarithms

In general: if $y = a^x$, then $x = \log_a y$

Write each of these equations using logarithm notation

$10^2 = 100$ $\log_{10} 100 = 2$

$5^2 = 25$ $\log_5 25 = 2$

$2^3 = 8$ $\log_2 8 = 3$

Write each of these equations using index notation

$\log_8 64 = 2$ $8^2 = 64$

$\log_2 32 = 5$ $2^5 = 32$

$\log_{27} 3 = \frac{1}{3}$ $27^{\frac{1}{3}} = 3$

WITHOUT USING A CALCULATOR...

Evaluate $\log_3 81$

$\log_3 81 = x \Leftrightarrow 3^x = 81$

$\Leftrightarrow 3^x = 3^4$

$\Leftrightarrow x = 4$

Answer: $\log_3 81 = 4$

Evaluate $\log_3\left(\frac{1}{9}\right)$

$\log_3\left(\frac{1}{9}\right) = x \rightarrow 3^x = \frac{1}{9}$

$\rightarrow 3^x = \frac{1}{3^2}$

$\rightarrow 3^x = 3^{-2}$

$\Leftrightarrow x = -2$

Answer: $\log_3\left(\frac{1}{9}\right) = -2$

Evaluate $\log_5\left(\frac{1}{25}\right)$

$\log_5\left(\frac{1}{25}\right) = x$ ☐ $5^x = \left(\frac{1}{25}\right)$

→ $5^x = \frac{1}{5^2}$

→ $5^x = 5^{-2}$

Answer: $\log_5\left(\frac{1}{25}\right) = -2$

Evaluate $\log_8\left(\frac{1}{4}\right)$

$\log_8\left(\frac{1}{4}\right) = x$ → $8^x = \frac{1}{4}$

→ $(2^3)^x = \frac{1}{2^2}$

→ $2^{3x} = 2^{-2}$

Answer: $\log_8\left(\frac{1}{4}\right) = \frac{-2}{3}$

5.1 The laws of logarithms

Exercise 14D

Exercise 5.1A Q1 and Q2

Find the value of:-

a) $\log_5 125^2$ b) $\log_5 \sqrt{125}$ c) $\log_5 \frac{1}{\sqrt{125}}$

Solve the equation $\log_5 125^x = 4$